ON PRESERVATION OF SPHERICAL SYMMETRY ON SPHERICAL GRID IN CARTESIAN COORDINATE SYSTEM IN COMPUTING GAS-DYNAMIC FLOWS USING EULER FINITE-VOLUME SCHEMES

N. A. Mikhailov, I. V. Glazyrin, N. L. Frolova, M. N. Chizhkov

FSUE «RFNC – VNIITF named after Academ. E. I. Zababakhin», Snezhinsk, Russia

The preservation of spherical flow symmetry is desirable in numerical schemes used for modeling the compression of inertial-confinement fusion (ICF) targets. This permits more precise research into physical causes of the disturbance of target compression symmetry, since it reduces the numerical contribution to the harmonic composition of perturbations of shell interfaces. The development of perturbations on interfaces, induced by hydrodynamic instabilities arising in the process of target compression, is the main reason that prevents thermonuclear ignition [1].

Much attention is given to the design of gas-dynamic schemes that preserve flow symmetry [2–4]. At present the Euler schemes are widely used for modeling IFC targets compression [5–7] due to the development of computing technologies and approaches to modeling multi-material flows on a fixed grid.

In order to preserve the flow symmetry, regardless of the mobility, the grid should be matched with the appropriate symmetry type [2]. In the case of spherical symmetry it should be radial. The simple version of such grid is a spherical one with the arrangement of nodes on coordinate surfaces of the spherical coordinate system. A separate issue is the choice of the coordinate system for recording the system of gas-dynamic equations. In a three-dimensional case the Cartesian coordinate system is the most preferable, since its system of equations has a divergence form suitable for providing the scheme conservatism. But in this case the faces of spherical grid cells are plane that can hinder the system to preserve the spherical symmetry.

The paper specifies sufficient conditions, so that finite-volume Euler schemes designed for computing gas-dynamic flows in Cartesian coordinate system, and using the Gaussian method for divergence and gradient operators, and the midpoint method for approximation integrals across cell faces, can preserve spherical symmetry on the spherical grid. Two approaches for providing the geometric condition on the relation of angular face areas to the cell volume are proposed, i.e. the correction of areas and special choice of polar angle partitioning. By way of example of symmetry preservation, if the sufficient conditions are fulfilled, the computation of the spherical problem on breakdown of a discontinuity according to the Godunov-type Euler scheme is considered [8].

References

1. **Ye Zhou.** Rayleigh–Taylor and Richtmyer–Meshkov instability induced flow, turbulence, and mixing. Part II. Physics Reports. – 2017. – Vol. 720–722. – P. 1–136.
2. **Margolin, L.** Using a curvilinear grid to construct symmetry-preserving discretizations for Lagrangian gas dynamics [Text] / L. Margolin, M. Shashkov // J. Comput. Phys. – 1999. – Vol. 149. – P. 389–417.
3. **Caramana, E. G.** A compatible, energy and symmetry preserving Lagrangian hydrodynamics algorithm in three-dimansional Cartisian geometry [Text] / E. G. Caramana, C. L. Rousculp, D. E. Burton // J. Comput. Phys. – 2000. – Vol. 157. – P. 89–119.
4. **Ling, D.** Positivity-preserving and symmetry-preserving Lagrangian schemes for compressible Euler equations in cylindrical coordinates [Text] / D. Ling, J. Cheng, C.-W. Shu // Computer and Fluids. – 2017. – Vol. 157. – P. 112–130.
5. **Getings, M.** The RAGE radiation-hydrodynamic code [Text] / M. Getings, R. Weaver, M. Clover et al. // Computational Science and Discovery. – 2008. – Vol. 1 (1).
6. **Fryxell, B.** FLASH: An adaptive mesh hydrodynamics code for modeling astrophysical thermonuclear flashes [Text] / B. Fryxell, K. Olson, P. Ricker et al. // Astrophysical Journal Supplement. – 2000. – Vol. 131. – P. 273–334.
7. **Lebo, I. G.** Investigation into hydrodynamic instabilities in laser thermonuclear synthesis problems by mathematical modeling methods [Text] / I. G. Lebo, V. F. Tishkin. – M. : FIZMATLIT, 2006. – P. 304.

8. **Glazyrin, I. V.** Finite-volume scheme for multicomponent compressible flows on unstructured grid in Focus three-dimensional complex [Text] / I. V. Glazyrin, N. A. Mikhailov // Vychislitelnaya matematica i matematicheskaya fizika [J. Computational Mathematics and Mathematical Physics]. – 2021. – Vol. 61, No. 6. – P. 1019–1033. (in Russian).